Suboptimality Estimation for the Semi-Discretized LQR Problem for Parabolic PDEs

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The linear quadratic regulator problem (LQR) for parabolic partial differential equations (PDEs) has been understood to be an infinite dimensional Hilbert space equivalent of the finite dimensional LQR problem known from mathematical systems theory. The matrix equations from the finite dimensional case become operator equations in the infinite dimensional Hilbert space setting. A rigorous convergence theory for the approximation of the infinite dimensional problem by Galerkin schemes in the space variable has been developed over the past decades. Numerical methods based on this approximation have been proven capable of solving the case of linear parabolic PDEs. Embedding these solvers in a model predictive control (MPC) scheme also nonlinear systems can be handled. Convergence rates for the approximation in the linear case are well understood in terms of the PDE's solution trajectories, as well as the solution operators of the underlying matrix/operator equations.

However, in practice engineers are often interested in suboptimality results in terms of the optimal cost, i.e., evaluation of the quadratic cost functional. In this contribution we are closing this gap in the theory.